

NICOLA FANELLI

CISE S.p.A. - Via Reggio Emilia, 39 - 20090 SEGRATE (MI)

PO Box 12081 - 20134 MILANO (ITALY)

## ABSTRACT

A new procedure, based on the representation of the noise properties of linear two-ports with uncorrelated noise waves, allows the experimental evaluation of the four noise parameters using only a sliding short as tunable element. Experimental set-up and results obtained from measurements of FET devices are discussed.

## INTRODUCTION

The improvement of GaAs FET performances requires accurate measurements of the device noise parameters at increasing frequencies. Conventional methods, based on the detection of the noise figure for at least four generator impedances, suffer from inaccuracies due to insufficient quality of the tuners employed in the measure, especially at frequencies above X-band. A recent procedure [1], based on the representation of the noise properties with two correlated noise waves, seems more adequate: it requires a generator with high reflection coefficient,  $|\Gamma_g| \approx 1$ , connected to the device input through a variable length of transmission line.

In this paper we will show that from the detection of the available noise power at device output, with a sliding short circuit connected at its input terminals, a relationship can be found between the device input impedance and the optimum source impedance for minimum noise figure,  $Z_{opt}$ . We were thus able to devise a simple procedure, using only a sliding short as tunable element, to determine the four noise parameters.

## THEORETICAL GROUNDS OF THE MEASURE.

The measurement method to be described is based on the representation of the noise properties of quadrupoles with uncorrelated noise waves placed at the input terminals [2,3]. The noise model, illustrated by the flow-graph in fig. 1a), considers an equivalent noise-free two-port, with scattering parameters  $[s_{ij}]$ , and two input noise sources,  $a_n, b_n$ ; the former to be added to the incident wave  $a_1$ , while the latter sums up with the reflected wave from the noise-free two-port to form  $b_1$ . Comparison with the Rothe-Dahlke noise model, fig. 1b), shows that the two noise sources,  $a_n, b_n$ , are given by

$$\begin{aligned} a_n &= -(v_n + z_1 i_n) / \sqrt{R_1} \\ b_n &= (v_n - z_1^* i_n) / \sqrt{R_1} \end{aligned} \quad (1)$$

where  $Z_1 = R_1 + jX_1$  is a normalization impedance.

Generally  $a_n$  and  $b_n$  are correlated, but it can be

shown that if the optimum source impedance for minimum noise figure,  $Z_{opt}$ , is chosen to normalize the waves at the input port, the two noise sources are uncorrelated. In this case, which we consider in the followings, a simple dependance of the noise figure  $F$ , from the source impedance,  $Z_s$ , is obtained [2] :

$$F = F_{min} + \alpha |\Gamma_s|^2 / (1 - |\Gamma_s|^2) \quad (2)$$

where

$$F_{min} = 1 + \frac{\langle a_n^2 \rangle}{kT_0}, \quad \alpha = \frac{\langle a_n^2 \rangle + \langle b_n^2 \rangle}{kT_0}, \quad \Gamma_s = \frac{Z_s - Z_{opt}}{Z_s + Z_{opt}^*},$$

$T_0$  is the room temperature and brackets indicate noise power per unit bandwidth. It can be shown that  $\langle a_n^2 \rangle$  and  $\langle b_n^2 \rangle$ , hence  $\alpha$ , are invariant for lossless transformation of the input reference plane.

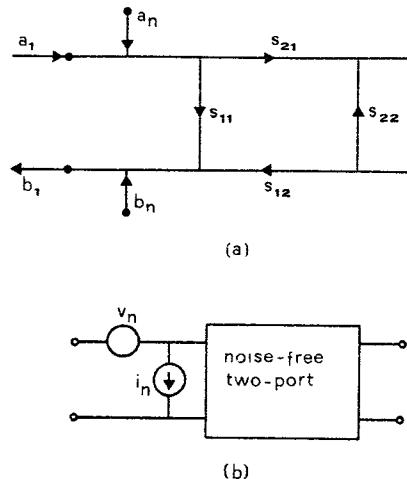


Fig. 1 - a) Noise wave model of a noisy two-port;  
b) Rothe-Dahlke noise model.

We examine now the noise power,  $\langle b_{2n}^2 \rangle$ , outgoing from the output terminals of the quadrupole when an arbitrary impedance  $Z_g$  is connected at the input. From the flow-graph in Fig. 2 we obtain:

$$\langle b_{2n}^2 \rangle = \frac{(1 - |\Gamma_g|^2)kT_0 + \langle a_n^2 \rangle + |\Gamma_g|^2 \langle b_n^2 \rangle}{|1 - s_{11} \Gamma_g|^2} |s_{21}|^2 \quad (3)$$

where  $\Gamma_g = (Z_g - Z_{opt}) / (Z_g + Z_{opt}^*)$ , and the scattering parameters of the device are normalized to  $Z_{opt}$  at the

input port and  $50 \Omega$  at the output. For  $|\Gamma_g| = 1$ :

$$\langle b_{2n}^2 \rangle = (\langle a_n^2 \rangle + \langle b_n^2 \rangle) \frac{|s_{21}|^2}{|1 - s_{11} \Gamma_g|^2} = kT_0 \alpha \frac{|s_{21}|^2}{|1 - s_{11} \Gamma_g|^2} \quad (4)$$

From (4) we observe that by varying the phase of  $\Gamma_g$ ,  $\langle b_{2n}^2 \rangle$  goes through maxima and minima whose rate

$r^2$  is:

$$r^2 = \frac{\langle b_{2n}^2 \rangle_{\max}}{\langle b_{2n}^2 \rangle_{\min}} = \left( \frac{1 + |s_{11}|}{1 - |s_{11}|} \right)^2 \quad (5)$$

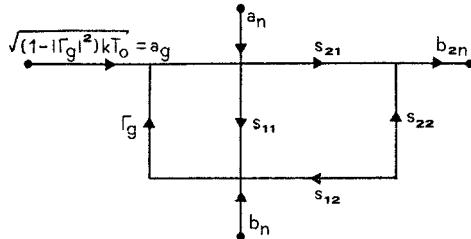


Fig. 2 - Determination of output noise power with arbitrary input termination.

Now adjust the sliding short to obtain a minimum value of  $\langle b_{2n}^2 \rangle$  and consider the relationship (4) at the reference plane where  $\Gamma_g = 1$ , i.e.  $Z_g = \infty$ . There we must have

$$s'_{11} \equiv \frac{z'_{in} - z'_{opt}^*}{z'_{in} + z'_{opt}} = -|s_{11}| \quad (6)$$

where  $z'_{in} = R_{in} + jX_{in}$  is the input impedance of the device, measured with a  $50 \Omega$  load at the output, and apices indicate quantities at the shifted reference plane. By solving (5), (6) with respect to  $z'_{opt} = R'_{opt} + jX'_{opt}$  we obtain:

$$\begin{aligned} R'_{opt} &= rR'_{in} \\ X'_{opt} &= -X'_{in} \end{aligned} \quad (7)$$

The value  $z'_{opt}$  can then be shifted back to the original reference plane to obtain  $Z_{opt}$ . We have thus established a procedure which relates the optimum source impedance for minimum noise figure to the input impedance of the device. Apparently, the problem of accurate determination of  $Z_{opt}$  has been reduced to the more manageable one of accurate measurement of the device input impedance. The relationship (7) is similar to usual assumptions for  $Z_{opt}$  of FET devices, with  $r$  ranging from 2 (X-band) to 1 (K-band) [4]. If these hypotheses hold true, a minimum of  $\langle b_{2n}^2 \rangle$  should be detected when the gate is open-circuited and in the last case,  $r=1$ ,  $\langle b_{2n}^2 \rangle$  should not ripple while sliding a short at the device input. The remaining

noise parameters  $\alpha$  and  $F_{\min}$  can be easily found once the value of  $Z_{opt}$  is known. By normalizing the input port to  $Z_{opt}$ ,  $|s_{21}|^2$  in (4) can be obtained from the measured forward gain in a  $50 \Omega$  system. Then we can apply (4) for any value of  $\Gamma_g$  to find out  $\alpha$ . Finally

$F_{\min}$  is determined from the measure of the noise figure with a  $50 \Omega$  generator impedance,  $F_{50}$ :

$$F_{\min} = F_{50} - \alpha \frac{|\Gamma_{50}|^2}{1 - |\Gamma_{50}|^2}$$

where  $|\Gamma_{50}| = |(Z_{opt} - 50)/(Z_{opt} + 50)|$ .

#### EXPERIMENTAL SET-UP AND RESULTS.

The output noise power from the DUT has been detected through the receiver apparatus of usual noise figure instrumentation. With a circulator at its input, the noise properties of the receiver are described by two noise waves like in fig. 3:  $b_r$  ( $\langle b_r^2 \rangle = kT_0$ ), is due to the thermal noise generated by the  $50 \Omega$  resistance seen when looking at the circulator input and  $a_r$  accounts for the noise internally generated in the receiver. Clearly the two terms are uncorrelated and application of relationship (2) would correctly give the receiver noise figure  $F_r$  as

$$F_r = F_r(50) / (1 - |\Gamma_s|^2)$$

where  $F_r(50) = 1 + \langle a_r^2 \rangle / kT_0$  is the minimum noise figure and  $\Gamma_s$  is the generator reflection coefficient.

$\langle a_r^2 \rangle$ , together with the gain of the receiving apparatus, are obtained through the measure of powers from a hot-cold noise source. From the flow-graph in fig. 3, the incident noise power at the receiver,  $P_n$ , will be:

$$P_n = \langle b_{2n}^2 \rangle + \langle a_r^2 \rangle + |s_{22} + \frac{s_{12}s_{21}\Gamma_g}{1 - s_{11}\Gamma_g}|^2 kT_0 \quad (8)$$

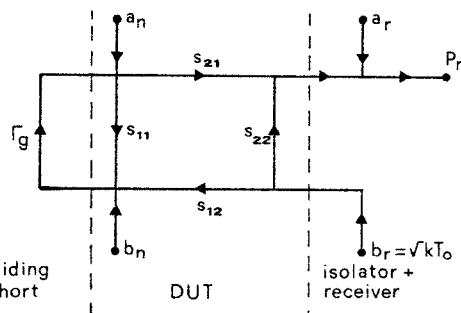


Fig. 3 - Flow-graph of the noise detection system.

In order to extract  $\langle b_{2n}^2 \rangle$  from (8), the last term should be evaluated. Although the scattering parameters and  $\Gamma_g$  in (8) depend on the unknown value of  $Z_{opt}$ , which normalizes waves at the input port, the coefficient of  $kT_0$  as a whole is the square modulus of the output reflection coefficient of DUT,  $\Gamma_2$ , when the input

is loaded with the sliding short. It is thus independent of the actual impedance value chosen to normalize the input port, and scattering parameters measured in  $50 \Omega$  system may be used to calculate  $\Gamma_2$ . We preferred instead to measure it directly for various positions of the input short: this avoids the errors associated with phase measurements of the various scattering parameters.

Typical results obtained at X-band, relative to 1 micron ALF-1020FETs, are listed in table 1. In order to increase the power transfer on a moderate bandwidth around 9.5 GHz, the devices were output matched on the microstrip carrier through the drain bond wire. A low frequency load was also provided through the drain bias circuit to avoid device instability. The sliding short employed in the measurements was in WR-90 waveguide. From table 1 we observe that  $R_{opt} \approx 3R_{in}$ , while  $X_{opt}$  is somewhat higher than  $-X_{in}$ . The degree of mismatch between  $Z_{opt}$  and  $Z_{in}$ , expressed by the factor  $r$ , clearly decreases with frequency. We also outline the smooth behaviour of  $\alpha$  versus frequency; this result should be compared with available data from commercial devices ( $\alpha = 4r_n g_{opt}$ ;  $r_n$  = noise resistance,  $g_{opt} = \text{Re}\{1/Z_{opt}\}$ ), where fluctuations wider than 200% are noticed for close frequencies. The preliminary results demonstrate that accurate estimates of the noise parameters, particularly  $Z_{opt}$  and  $\alpha$ , are achievable with the proposed method. Since the procedure requires a simple and high quality microwave device, the sliding short, and only one vector measurement, that of  $s_{11}$ , applicability is foreseeable at frequencies well above X-band.

TABLE 1

F(GHz)	r	$\alpha$	Fmin	Zopt( $\Omega$ )		Zin( $\Omega$ )	
				$R_{opt}$	$X_{opt}$	$Z_{in}$	$X_{in}$
8	4.9	1.1	2.5	33	54	8	-45
8.5	4.7	.9	2.4	32	51	8.5	-41
9	4.4	1.3	2.7	30	50.5	9	-38
9.5	4.1	1.2	3	35	48	10	-39
10	3.4	1.5	2.7	32	45	11	-37
10.5	3	1.8	3.2	32	42	13	-32.5
11	2.8	1.7	3.2	30	42.5	12.5	-32

#### ACKNOWLEDGEMENTS

The author thanks E.M. Bastida and V. Piemontese for helpful discussions and suggestions.

#### REFERENCES

- (1) R.P. Meys, "A new approach to the noise properties of linear microwave devices", IEEE tr. MTT-26, pp. 34-37, Jan. 1978.
- (2) P. Penfield, "Wave representation of amplifier noise", IRE tr. on Circuit Theory, pp. 84-86, March 1962.
- (3) H. Bauer, H. Rothe, "Der äquivalente Rauschvierpol als Wellenvierpol", Arch. elect. Übertragung, vol. 10, pp. 241-252, 1956.
- (4) A. Cappello, J. Pierro, "A 22 to 24 GHz cryogenically cooled low noise FET amplifier", 1982 IEEE MTT-S. symp. dig., p. 20.